Section 1.2 Analyzing Graphs of Functions

1. ordered pairs

3. zeros

5. maximum

7. odd

9. Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \infty)$

11. Domain: $[-4, 4]$

Range: $[0, 4]$

13. Domain: $(-\infty, \infty)$; Range: $[-4, \infty)$

(a) $f(-2) = 0$

(b) $f(-1) = -1$

(c) $f\left(\frac{1}{2}\right) = 0$

(d) $f(1) = -2$

15. Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

(a) $f(2) = 0$

(b) $f(1) = 1$

(c) $f(3) = 2$

(d) $f(-1) = 3$

17. $y = \frac{1}{2}x^2$

A vertical line intersects the graph at most once, so y *is* a function of x.

19. $x - y^2 = 1 \Rightarrow y = \pm\sqrt{x - 1}$

y *is not* a function of x. Some vertical lines intersect the graph twice.

21. $x^2 = 2xy - 1$

A vertical line intersects the graph at most once, so y *is* a function of x.

23. $f(x) = 2x^2 - 7x - 30$

$2x^2 - 7x - 30 = 0$

$(2x + 5)(x - 6) = 0$

$2x + 5 = 0$ or $x - 6 = 0$

$x = -\frac{5}{2}$ or $x = 6$

25. $f(x) = \dfrac{x}{9x^2 - 4}$

$\dfrac{x}{9x^2 - 4} = 0$

$x = 0$

27. $f(x) = \frac{1}{2}x^3 - x$

$\frac{1}{2}x^3 - x = 0$

$x^3 - 2x = 2(0)$

$x(x^2 - 2) = 0$

$x = 0$ or $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

29. $f(x) = 4x^3 - 24x^2 - x + 6$

$4x^3 - 24x^2 - x + 6 = 0$

$4x^2(x - 6) - 1(x - 6) = 0$

$(x - 6)(4x^2 - 1) = 0$

$(x - 6)(2x + 1)(2x - 1) = 0$

$x - 6 = 0, \quad 2x + 1 = 0, \quad 2x - 1 = 0$

$x = 6, \quad x = -\frac{1}{2}, \quad x = \frac{1}{2}$

31. $f(x) = \sqrt{2x} - 1$

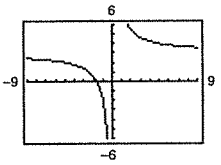
$\sqrt{2x} - 1 = 0$

$\sqrt{2x} = 1$

$2x = 1$

$x = \frac{1}{2}$

33. (a)



Zero: $x = -\dfrac{5}{3}$

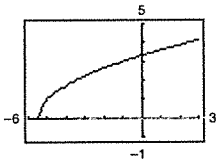
(b) $f(x) = 3 + \dfrac{5}{x}$

$3 + \dfrac{5}{x} = 0$

$3x + 5 = 0$

$x = -\dfrac{5}{3}$

35. (a)



Zero: $x = -\frac{11}{2}$

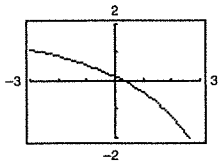
(b) $f(x) = \sqrt{2x + 11}$

$\sqrt{2x + 11} = 0$

$2x + 11 = 0$

$x = -\frac{11}{2}$

37. (a)



Zero: $x = \dfrac{1}{3}$

(b) $f(x) = \dfrac{3x - 1}{x - 6}$

$\dfrac{3x - 1}{x - 6} = 0$

$3x - 1 = 0$

$x = \dfrac{1}{3}$

39. $f(x) = \frac{3}{2}x$

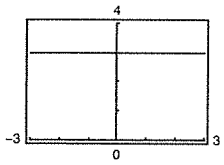
The function is increasing on $(-\infty, \infty)$.

41. $f(x) = x^3 - 3x^2 + 2$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

43. $f(x) = 3$

(a)



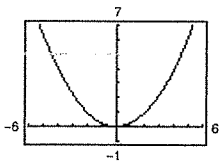
Constant on $(-\infty, \infty)$

(b)

x	-2	-1	0	1	2
$f(x)$	3	3	3	3	3

45. $g(s) = \dfrac{s^2}{4}$

(a)



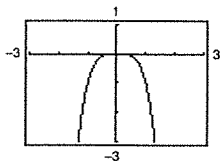
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

s	-4	-2	0	2	4
$g(s)$	4	1	0	1	4

47. $f(t) = -t^4$

(a)



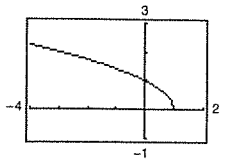
Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$

(b)

t	-2	-1	0	1	2
$f(t)$	-16	-1	0	-1	-16

49. $f(x) = \sqrt{1 - x}$

(a)



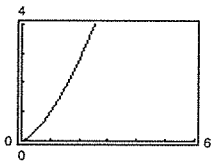
Decreasing on $(-\infty, 1)$

(b)

x	-3	-2	-1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

51. $f(x) = x^{3/2}$

(a)



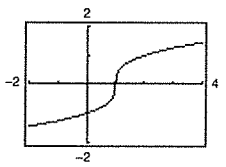
Increasing on $(0, \infty)$

(b)

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

53. $g(t) = \sqrt[3]{t - 1}$

(a)



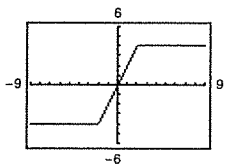
Increasing on $(-\infty, \infty)$

(b)

t	-2	-1	0	1	2
$g(t)$	-1.44	-1.26	-1	0	1

55. $f(x) = |x + 2| - |x - 2|$

(a)



Increasing on $(-2, 2)$

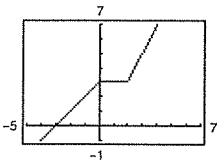
Constant on $(-\infty, -2)$ and $(2, \infty)$

(b)

x	-2	-1	0	1	4
$f(x)$	-4	-2	0	2	4

57. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x - 1, & x > 2 \end{cases}$

(a)



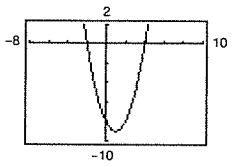
Increasing on $(-\infty, 0)$ and $(2, \infty)$

Constant on $(0, 2)$

(b)

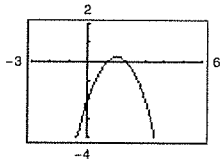
x	-2	-1	0	1	2	3	4
$f(x)$	1	2	3	3	3	5	7

59. $f(x) = (x - 4)(x + 2)$



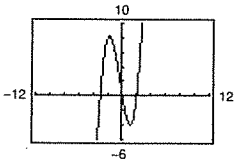
Relative minimum: $(1, -9)$

61. $f(x) = -x^2 + 3x - 2$



Relative maximum: $(1.5, 0.25)$

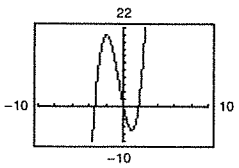
63. $f(x) = x(x - 2)(x + 3)$



Relative minimum: $(1.12, -4.06)$

Relative maximum: $(-1.79, 8.21)$

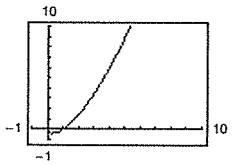
65.



Relative minimum: $(1, -7)$

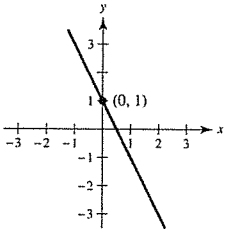
Relative maximum: $(-2, 20)$

67.

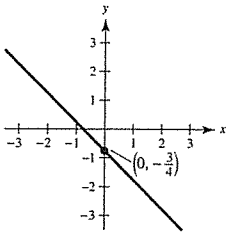


Relative minimum: $(0.33, -0.38)$

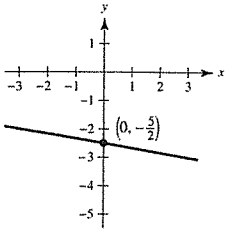
69. $f(x) = 1 - 2x$



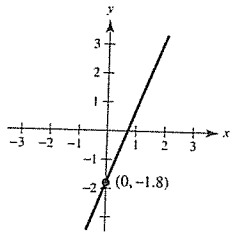
71. $f(x) = -x - \frac{3}{4}$



73. $f(x) = -\frac{1}{6}x - \frac{5}{2}$



75. $f(x) = -1.8 + 2.5x$



77. (a) $f(1) = 4, f(0) = 6$

$(1, 4), (0, 6)$

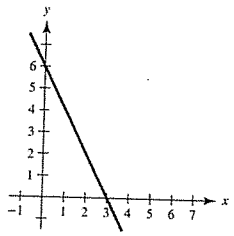
$$m = \frac{6 - 4}{0 - 1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$

(b)



79. (a) $f(5) = -4, f(-2) = 17$

$(5, -4), (-2, 17)$

$$m = \frac{17 - (-4)}{-2 - 5} = \frac{21}{-7} = -3$$

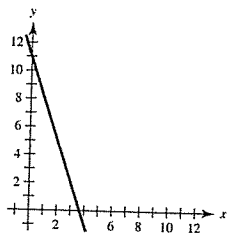
$$y - (-4) = -3(x - 5)$$

$$y + 4 = -3x + 15$$

$$y = -3x + 11$$

$$f(x) = -3x + 11$$

(b)



81. (a) $f(-5) = -1, f(5) = -1$

$(-5, -1), (5, -1)$

$$m = \frac{-1 - (-1)}{5 - (-5)} = \frac{0}{10} = 0$$

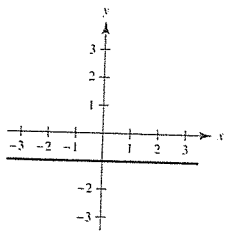
$$y - (-1) = 0(x - (-5))$$

$$y + 1 = 0$$

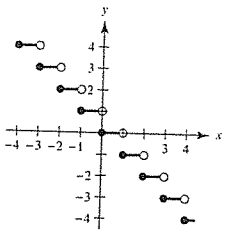
$$y = -1$$

$$f(x) = -1$$

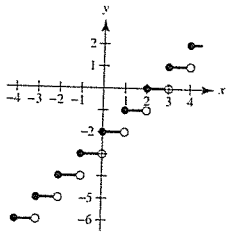
(b)



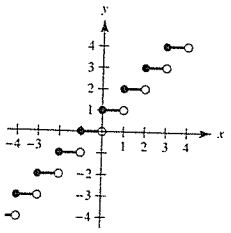
83. $g(x) = -[\![x]\!]$



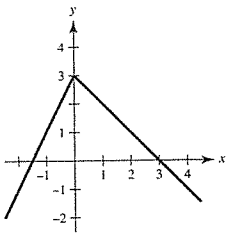
85. $g(x) = [\![x]\!] - 2$



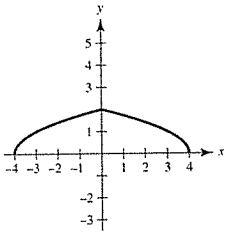
87. $g(x) = [\![x + 1]\!]$



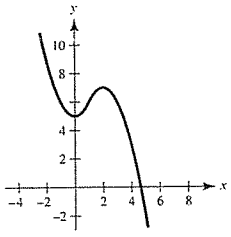
89. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$



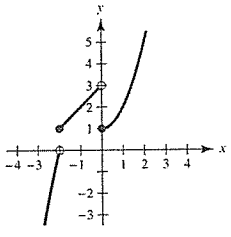
91. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



93. $f(x) = \begin{cases} x^2 + 5, & x \leq 1 \\ -x^2 + 4x + 3, & x > 1 \end{cases}$

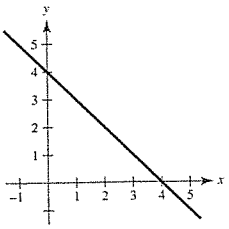


95. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \leq x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

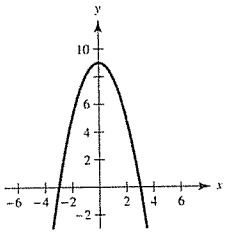


97. $f(x) = 4 - x$

$f(x) \geq 0$ on $(-\infty, 4]$.



99. $f(x) = 9 - x^2$



$f(x) \geq 0$ on $[-3, 3]$

101. $f(x) = \sqrt{x - 1}$

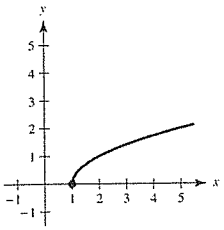
$f(x) \geq 0$ on $[1, \infty)$.

$\sqrt{x - 1} \geq 0$

$x - 1 \geq 0$

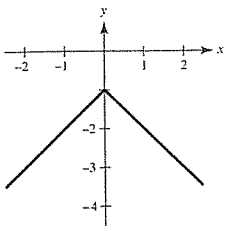
$x \geq 1$

$[1, \infty)$



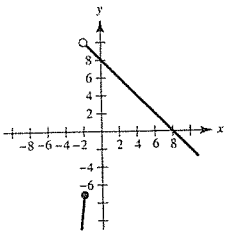
103. $f(x) = -(1 + |x|)$

$f(x)$ is never greater than 0. $(f(x) < 0$ for all $x.)$



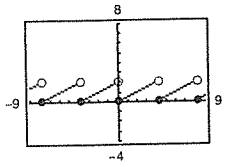
105. $f(x) = \begin{cases} 1 - 2x^2, & x \le -2 \\ -x + 8, & x > -2 \end{cases}$

$f(x) \ge 0$ on $(-2, 8]$



107. $s(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)$

(a)



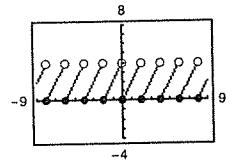
(b) Domain: $(-\infty, \infty)$

Range: $[0, 2)$

(c) Sawtooth pattern

109. $h(x) = 4\left(\frac{1}{2}x - \left[\!\left[\frac{1}{2}x\right]\!\right]\right)$

(a)

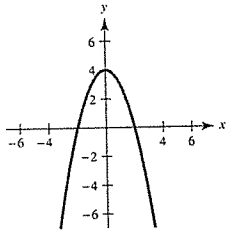


(b) Domain: $(-\infty, \infty)$

Range: $[0, 4)$

(c) Sawtooth pattern

111.



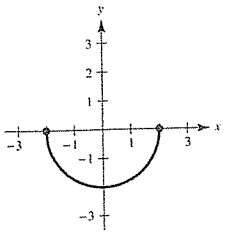
(a) Domain: All real numbers or $(-\infty, \infty)$

(b) Range: $(-\infty, 4]$

(c) Increasing on $(-\infty, 0)$

Decreasing on $(0, \infty)$

113.



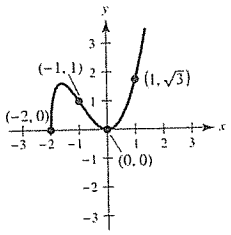
(a) Domain: $[-2, 2]$

(b) Range: $[-2, 0]$

(c) Increasing on $(0, 2)$

Decreasing on $(-2, 0)$

115.



(a) $f(-1) = 1$

(b) $f(1) = \sqrt{3}$

(c) f is increasing on $(-2, -1.6)$ and $(0, \infty)$

f is decreasing on $(-1.6, 0)$.

117. $f(x) = x^6 - 2x^2 + 3$

$f(-x) = (-x)^6 - 2(-x)^2 + 3$

$= x^6 - 2x^2 + 3$

$= f(x)$

The function is even. y-axis symmetry.

119. $g(x) = x^3 - 5x$

$g(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -g(x)$

The function is odd. Origin symmetry.

121. $h(x) = x\sqrt{x + 5}$

$h(-x) = (-x)\sqrt{-x + 5}$

$= -x\sqrt{5 - x}$

$\ne h(x)$

$\ne -h(x)$

The function is neither odd nor even. No symmetry

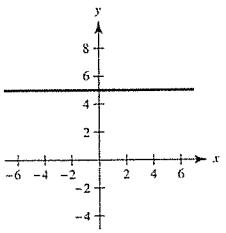
123. $f(s) = 4s^{3/2}$

$= 4(-s)^{3/2}$

$\neq f(s)$

$\neq -f(s)$

125.



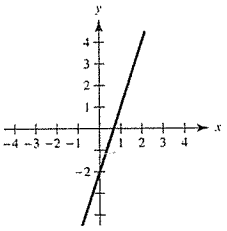
The graph of $f(x) = 5$ is symmetric to the y-axis, which implies $f(x)$ is even.

$f(-x) = 5$

$= f(x)$

The function is even.

127. $f(x) = 3x - 2$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(-x) = 3(-x) - 2$

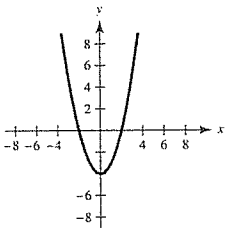
$= -3x - 2$

$\neq f(x)$

$\neq -f(x)$

The function is neither even nor odd.

129. $h(x) = x^2 - 4$

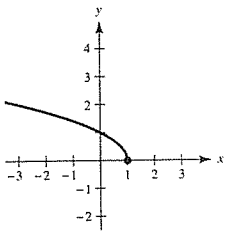


The graph displays y-axis symmetry, which implies $h(x)$ is even.

$h(-x) = (-x)^2 - 4 = x^2 - 4 = h(x)$

The function is even.

131. $f(x) = \sqrt{1 - x}$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(-x) = \sqrt{1 - (-x)}$

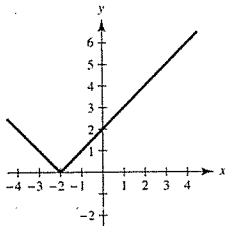
$= \sqrt{1 + x}$

$\neq f(x)$

$\neq -f(x)$

The function is neither even nor odd.

133. $f(x) = |x + 2|$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(-x) = |-x + 2| \neq f(x) \neq -f(x)$

The function is neither even nor odd.

135. h = top − bottom

$= (-x^2 + 4x - 1) - 2$

$= -x^2 + 4x - 3$

137. h = top − bottom

$= (4x - x^2) - 2x$

$= 2x - x^2$

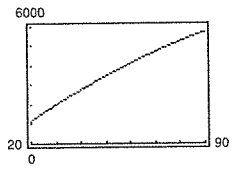
139. L = right − left

$= \frac{1}{2}y^2 - 0 = \frac{1}{2}y^2$

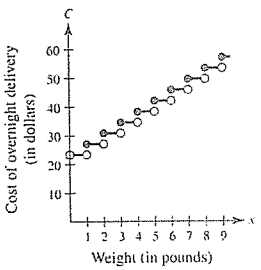
141. L = right − left

$= 4 - y^2$

143. $L = -0.294x^2 + 97.744x - 664.875, 20 \le x \le 90$

(a) 

(b) $L = 2000$ when $x \approx 29.9645 \approx 30$ watts.

145. (a) 

(b) $C(9.25) = 23.40 + 3.75[\![9.25]\!]$
$= 23.40 + 3.75(9)$
$= 57.15$

It costs \$57.15 to mail a 9.25 pound package.

147. (a) For the average salaries of college professors, a scale of \$10,000 would be appropriate.

(b) For the population of the United States, use a scale of 10,000,000.

(c) For the percent of the civilian workforce that is unemployed, use a scale of 1%.

149. False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

151. (a) Even. The graph is a reflection in the x-axis.

(b) Even. The graph is a reflection in the y-axis.

(c) Even. The graph is a vertical translation of f.

(d) Neither. The graph is a horizontal translation of f.

153. $\left(-\frac{3}{2}, 4\right)$

(a) If f is even, another point is $\left(\frac{3}{2}, 4\right)$.

(b) If f is odd, another point is $\left(\frac{3}{2}, -4\right)$.

155. $(4, 9)$

(a) If f is even, another point is $(-4, 9)$.

(b) If f is odd, another point is $(-4, -9)$.

157. (a) $(-x, -y)$

(b) $(\quad, \quad)$

159. $f(x) = \begin{cases} x + 2, & x < -2 \\ 0, & -2 \le x \le 2 \\ ax + b, & x > 2 \end{cases}$

(a) If $f(x)$ is odd, then $f(-x) = -f(x)$

$f(-3) = -f(3) \Rightarrow -1 = -(3a + b)$

$f(-4) = -f(4) \Rightarrow -2 = -(4a + b)$

Solving the system: $4a + b = 2$

$3a + b = 1$

yields $a = 1$ and $b = -2$

(b) If $f(x)$ is even, then $f(-x) = f(x)$

$f(-3) = f(3) \Rightarrow -1 = 3a + b$

$f(-4) = f(4) \Rightarrow -2 = 4a + b$

Solving this system yields $a = -1$ and $b = 2$.