CHAPTER 1
Functions and Their Graphs

Section 1.1 Functions

1. domain; range; function

3. independent; dependent

5. implied domain

7. Yes, the relationship is a function. Each domain value is matched with exactly one range value.

9. No, the relationship is not a function. The domain values are each matched with two range values.

11. No, the relationship is not a function. The domain values are each matched with three range values.

13. Yes, it does represent a function. Each input value is matched with exactly one output value.

15. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

17. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
    (b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.
    (c) Each element of A is matched with exactly one element of B, so it does represent a function.
    (d) The element 2 in A is not matched with an element of B, so the relation does not represent a function.

19. Each is a function. For each year there corresponds one and only one circulation.

21. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

    No, y *is not* a function of x.

23. $x^2 + y = 4 \Rightarrow y = 4 - x^2$

    Yes, y *is* a function of x.

25. $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$

    Yes, y *is* a function of x.

27. $(x + 2)^2 + (y - 1)^2 = 25$

    $$y = \pm\sqrt{25 - (x + 2)^2} + 1$$

    No, y *is not* a function of x.

29. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

    No, y *is not* a function of x.

31. $y = \sqrt{16 - x^2}$

    Yes, y *is* a function of x.

33. $y = |4 - x|$

    Yes, y *is* a function of x.

35. $x = 14$

    No, this *is not* a function of x.

37. $y + 5 = 0$

    $y = -5$ or $y = 0x - 5$

    Yes, y *is* a function of x.

39. $f(x) = 2x - 3$
    (a) $f(1) = 2(1) - 3 = -1$
    (b) $f(-3) = 2(-3) - 3 = -9$
    (c) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

41. $V(r) = \frac{4}{3}\pi r^3$
    (a) $V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$
    (b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right) = \frac{9}{2}\pi$
    (c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

43. $g(t) = 4t^2 - 3t + 5$
    (a) $g(2) = 4(2)^2 - 3(2) + 5$
        $= 15$
    (b) $g(t - 2) = 4(t - 2)^2 - 3(t - 2) + 5$
        $= 4t^2 - 19t + 27$
    (c) $g(t) - g(2) = 4t^2 - 3t + 5 - 15$
        $= 4t^2 - 3t - 10$

45. $f(y) = 3 - \sqrt{y}$
    (a) $f(4) = 3 - \sqrt{4} = 1$
    (b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$
    (c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

47. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

49. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x - 1) = \dfrac{|x - 1|}{x - 1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}$

51. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

53. $f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$

(a) $f(-2) = 3(-2) - 1 = -7$

(b) $f\left(-\frac{1}{2}\right) = 4$

(c) $f(3) = 3^2 = 9$

55. $f(x) = x^2 - 3$

$f(-2) = (-2)^2 - 3 = 1$

$f(-1) = (-1)^2 - 3 = -2$

$f(0) = (0)^2 - 3 = -3$

$f(1) = (1)^2 - 3 = -2$

$f(2) = (2)^2 - 3 = 1$

x	-2	-1	0	1	2
$f(x)$	1	-2	-3	-2	1

57. $h(t) = \frac{1}{2}|t + 3|$

$h(-5) = \frac{1}{2}|-5 + 3| = 1$

$h(-4) = \frac{1}{2}|-4 + 3| = \frac{1}{2}$

$h(-3) = \frac{1}{2}|-3 + 3| = 0$

$h(-2) = \frac{1}{2}|-2 + 3| = \frac{1}{2}$

$h(-1) = \frac{1}{2}|-1 + 3| = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

59. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x - 2)^2, & x > 0 \end{cases}$

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1 - 2)^2 = 1$

$f(2) = (2 - 2)^2 = 0$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

61. $15 - 3x = 0$

$3x = 15$

$x = 5$

63. $\dfrac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \dfrac{4}{3}$

65. $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

67. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0, x = -1,$ or $x = 1$

69.
$$\begin{aligned} f(x) &= g(x) \\ x^2 &= x + 2 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \end{aligned}$$
$$x - 2 = 0 \quad x + 1 = 0$$
$$x = 2 \qquad x = -1$$

71.
$$\begin{aligned} f(x) &= g(x) \\ x^4 - 2x^2 &= 2x^2 \\ x^4 - 4x^2 &= 0 \\ x^2(x^2 - 4) &= 0 \\ x^2(x + 2)(x - 2) &= 0 \\ x^2 &= 0 \Rightarrow x = 0 \\ x + 2 &= 0 \Rightarrow x = -2 \\ x - 2 &= 0 \Rightarrow x = 2 \end{aligned}$$

73. $f(x) = 5x^2 + 2x - 1$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

75. $h(t) = \dfrac{4}{t}$

The domain is all real numbers t except $t = 0$.

77. $g(y) = \sqrt{y - 10}$

Domain: $y - 10 \geq 0$

$y \geq 10$

The domain is all real numbers y such that $y \geq 10$.

79. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

The domain is all real numbers x except $x = 0, x = -2$.

81. $f(s) = \dfrac{\sqrt{s - 1}}{s - 4}$

Domain: $s - 1 \geq 0 \Rightarrow s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

83. $f(x) = \dfrac{x - 4}{\sqrt{x}}$

The domain is all real numbers such that $x > 0$ or $(0, \infty)$.

85.
$$\begin{aligned} f(x) &= x^2 \\ f(-2) &= (-2)^2 = 4 \\ f(-1) &= (-1)^2 = 1 \\ f(0) &= 0^2 = 0 \\ f(1) &= 1^2 = 1 \\ f(2) &= 2^2 = 4 \end{aligned}$$
$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

87.
$$\begin{aligned} f(x) &= |x| + 2 \\ f(-2) &= |-2| + 2 = 4 \\ f(-1) &= |-1| + 2 = 3 \\ f(0) &= |0| + 2 = 2 \\ f(1) &= |1| + 2 = 3 \\ f(2) &= |2| + 2 = 4 \end{aligned}$$
$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

89. No. The element 3 in the domain corresponds to two elements in the range.

91. $A = s^2$ and $P = 4s \Rightarrow \dfrac{P}{4} = s$

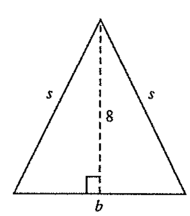
$$A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$$

93.
$$\begin{aligned} 8^2 + \left(\frac{b}{2}\right)^2 &= s^2 \\ \frac{b^2}{4} &= s^2 - 64 \\ b^2 &= 4(s^2 - 64) \\ b &= 2\sqrt{s^2 - 64} \end{aligned}$$

Thus, $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}\left(2\sqrt{s^2 - 64}\right)(8)$

$= 8\sqrt{s^2 - 64}$ square inches.

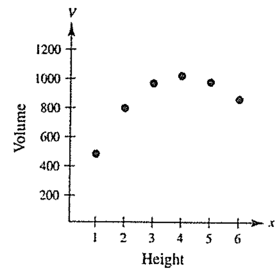


95. (a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



V is a function of x.

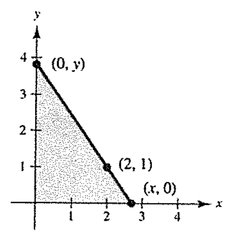
(c) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

97. $A = \frac{1}{2}bh = \frac{1}{2}xy$

Because $(0, y), (2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$\frac{1-y}{2-0} = \frac{0-1}{x-2}$$
$$\frac{1-y}{2} = \frac{-1}{x-2}$$
$$y = \frac{2}{x-2} + 1$$
$$y = \frac{x}{x-2}$$



So, $A = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2(x-2)}$.

The domain of A includes x-values such that $x^2/\left[2(x-2)\right] > 0$. By solving this inequality, the domain is $x > 2$.

99. $y = -\frac{1}{10}x^2 + 3x + 6$

$y(30) = -\frac{1}{10}(30)^2 + 3(30) + 6 = 6$ feet

If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 6 feet

101. $p(t) = \begin{cases} 1.011t^2 - 12.38t + 170.5, & 8 \le t \le 13 \\ -6.950t^2 + 222.55t - 1557.6, & 14 \le t \le 17 \end{cases}$

1998: Use $t = 8$ and find $p(8)$. $\quad p(8) = 1.011(8)^2 - 12.38(t) + 170.5 = 136.164$ thousand $= \$136,164$

1999: Use $t = 9$ and find $p(9)$. $\quad p(9) = 1.011(9)^2 - 12.38(9) + 170.5 = 140.971$ thousand $= \$140,971$

2000: Use $t = 10$ and find $p(10)$. $\quad p(10) = 1.011(10)^2 - 12.38(10) + 170.5 = 147.800$ thousand $= \$147,800$

2001: Use $t = 11$ and find $p(11)$. $\quad p(11) = 1.011(11)^2 - 12.38(11) + 170.5 = 156.651$ thousand $= \$156,651$

2002: Use $t = 12$ and find $p(12)$. $\quad p(12) = 1.011(12)^2 - 12.38(12) + 170.5 = 167.524$ thousand $= \$167,524$

2003: Use $t = 13$ and find $p(13)$. $\quad p(13) = 1.011(13)^2 - 12.38(13) + 170.5 = 180.419$ thousand $= \$180,419$

2004: Use $t = 14$ and find $p(14)$. $\quad p(14) = -6.950(14)^2 + 222.55(14) - 1557.6 = 195.900$ thousand $= \$195,900$

2005: Use $t = 15$ and find $p(15)$. $\quad p(15) = -6.950(15)^2 + 222.55(15) - 1557.6 = 216.900$ thousand $= \$216,900$

2006: Use $t = 16$ and find $p(16)$. $\quad p(16) = -6.950(16)^2 + 222.55(16) - 1557.6 = 224.000$ thousand $= \$224,000$

2007: Use $t = 17$ and find $p(17)$. $\quad p(17) = -6.950(17)^2 + 222.55(17) - 1557.6 = 217.200$ thousand $= \$217,200$

103. (a) Cost = variable costs + fixed costs

$C = 12.30x + 98{,}000$

(b) Revenue = price per unit × number of units

$R = 17.98x$

(c) Profit = Revenue − Cost

$P = 17.98x - (12.30x + 98{,}000)$

$P = 5.68x - 98{,}000$

105. (a) $R = n(\text{rate}) = n[8.00 - 0.05(n - 80)],\ n \ge 80$

$R = 12.00n - 0.05n^2 = 12n - \dfrac{n^2}{20} = \dfrac{240n - n^2}{20},\ n \ge 80$

(b)

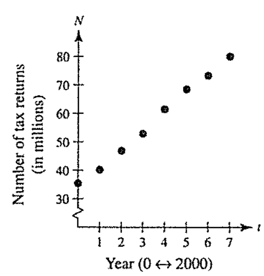
n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

107. (a) $\dfrac{f(2007) - f(2000)}{2007 - 2000} = \dfrac{80.0 - 35.4}{2007 - 2000} \approx 6.37$

Approximately 6.37 million more tax returns were made through e-file each year from 2000 to 2007.

(b)



(c) Use the points $(0, 35.4)$ and $(7, 80.0)$.

$m = \dfrac{80.0 - 35.4}{7 - 0} = 6.37$

$N = 6.37t + 35.4$

(d)

t	0	1	2	3	4	5	6	7
N	35.4	41.8	48.1	54.5	60.9	67.3	73.6	80.0

(e) Using a graphing utility yields the model $N = 6.56t + 34.4$. Compared to the model in part (c), the model generated by the graphing utility produces values that reflect the data more accurately.

109.

$$\begin{aligned} f(x) &= x^2 - x + 1 \\ f(2 + h) &= (2 + h)^2 - (2 + h) + 1 \\ &= 4 + 4h + h^2 - 2 - h + 1 \\ &= h^2 + 3h + 3 \\ f(2) &= (2)^2 - 2 + 1 = 3 \\ f(2 + h) - f(2) &= h^2 + 3h \\ \frac{f(2 + h) - f(2)}{h} &= \frac{h^2 + 3h}{h} = h + 3,\ h \ne 0 \end{aligned}$$

111. $f(x) = x^3 + 2x - 1$

$$\frac{f(x+c) - f(x)}{c} = \frac{\left[(x+c)^3 + 2(x+c) - 1\right] - \left(x^3 + 2x - 1\right)}{c}$$
$$= \frac{x^3 + 3x^2c + 3xc^2 + c^3 + 2x + 2c - 1 - x^3 - 2x + 1}{c}$$
$$= \frac{3x^2c + 3xc^2 + c^3 + 2c}{c} = \frac{c\left(3x^2 + 3xc + c^2 + 2\right)}{c}$$
$$= 3x^2 + 3xc + c^2 + 2, \quad c \neq 0$$

113. $g(x) = 3x - 1$

$$\frac{g(x) - g(3)}{x - 3} = \frac{(3x - 1) - 8}{x - 3} = \frac{3x - 9}{x - 3} = \frac{3(x - 3)}{x - 3} = 3, x \neq 3$$

115. $f(x) = \sqrt{5x}$

$$\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{5x} - 5}{x - 5}$$

117. By plotting the points, we have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

119. Because the function is undefined at 0, we have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

121. False. The equation $y^2 = x^2 + 4$ is a relation between x and y. However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

123. True.

As long as **all** elements in the domain are matched with elements in the range, even if it is the same element, then

125. False. The range is $[-1, \infty)$.

127. $f(x) = \sqrt{x - 1}$ Domain: $x \geq 1$

$g(x) = \dfrac{1}{\sqrt{x - 1}}$ Domain: $x > 1$

The value 1 may be included in the domain of $f(x)$ as it is possible to find the square root of 0. However, 1 cannot be included in the domain of $g(x)$ as it causes a zero to occur in the denominator which results in the function being undefined.

129. No; x is the independent variable, f is the name of the function.

131. (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.